Generalized Fuzzy Password-Authenticated Key Exchange from Error Correcting Codes

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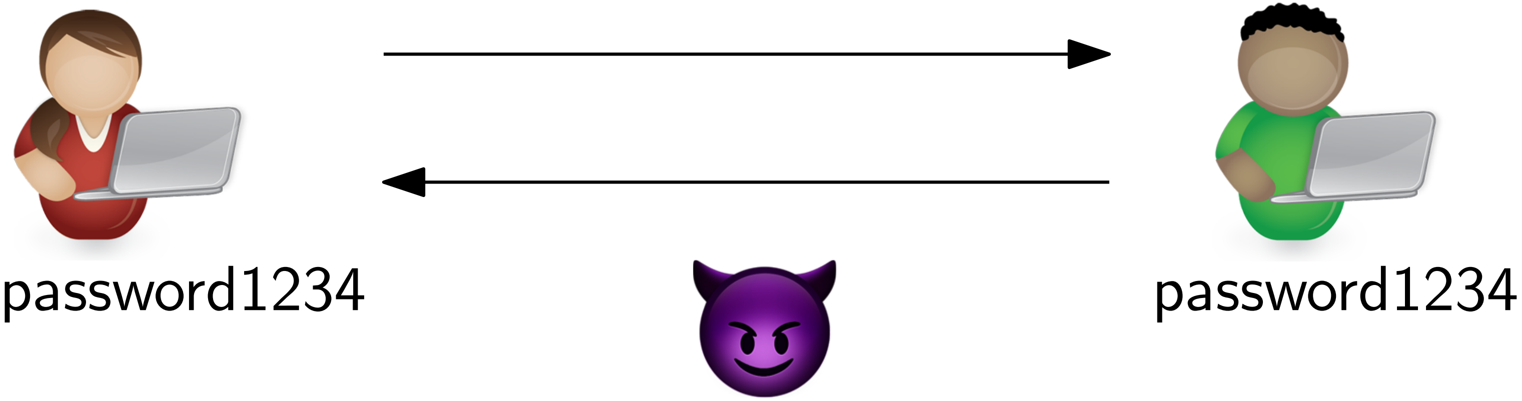
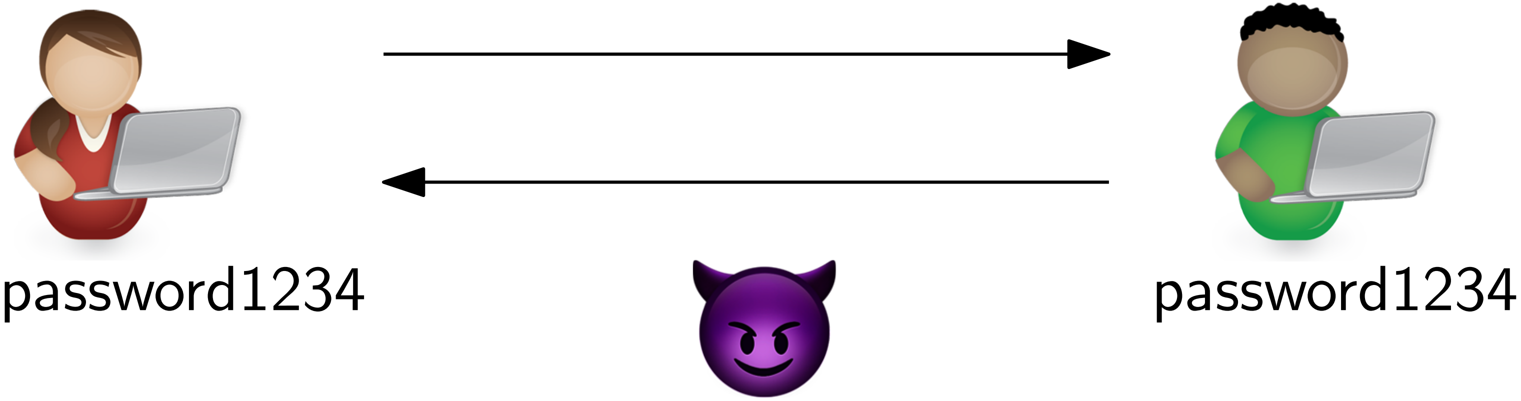
Password Authenticated Key Exchange (PAKE)



password1234

password1234

Password Authenticated Key Exchange (PAKE)



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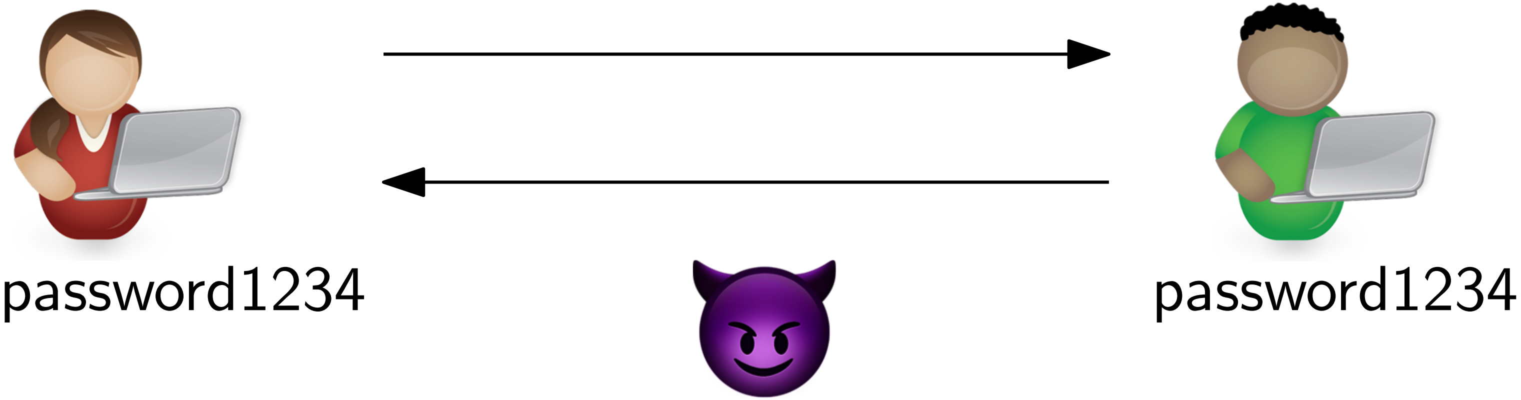
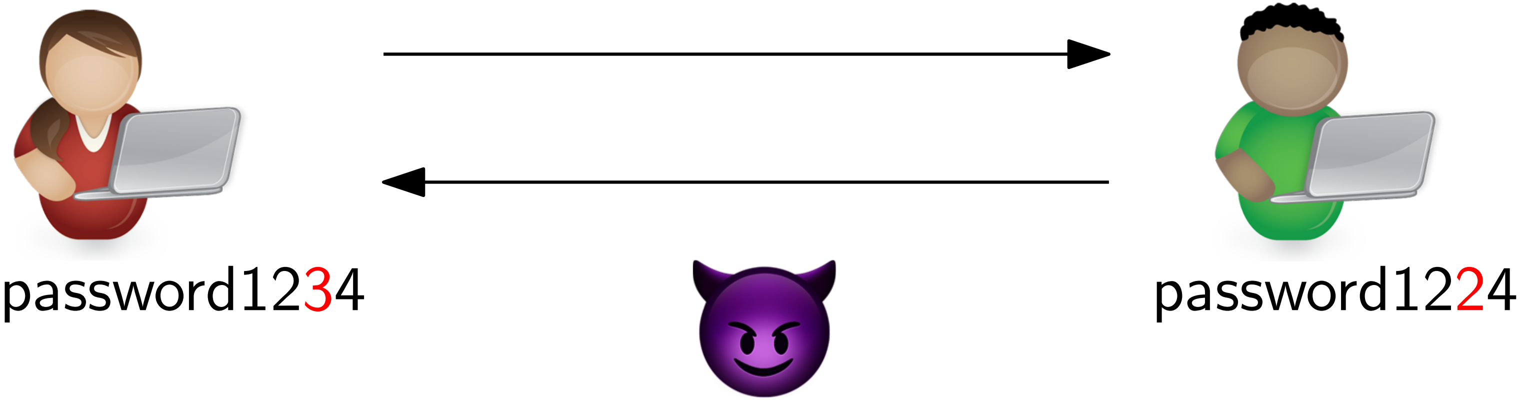


password1234

password1234

Fuzzy PAKE

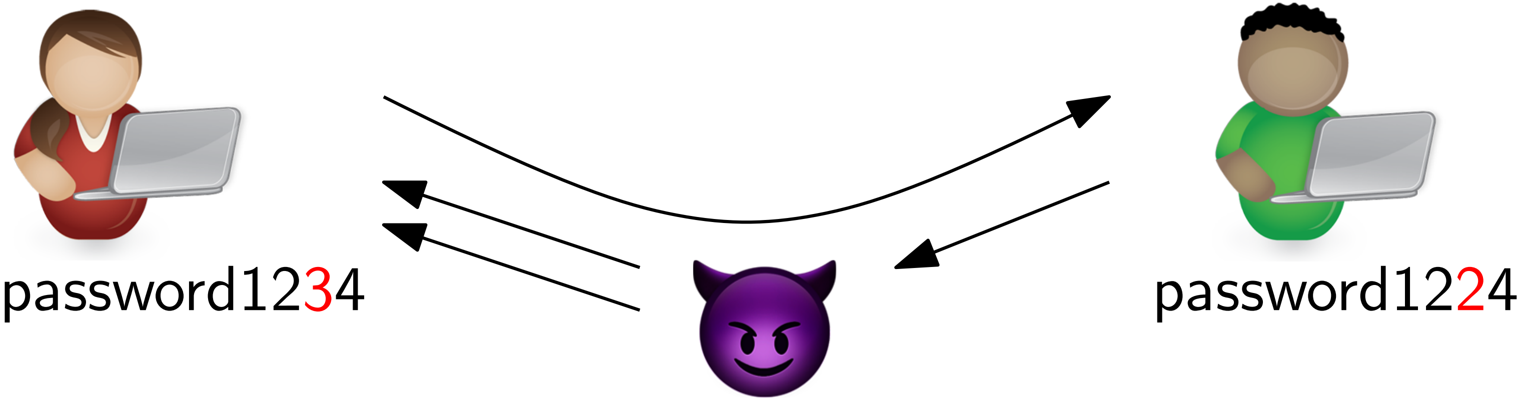


password1234

password1224

- Typo tolerance
- Biometric readings as passwords
- Pairing of nearby IoT devices with sensor data as passwords

Fuzzy PAKE - Attack Model



password1234

password1224

- Same session-key, if and only if the passwords are close
- $\mathcal{A}$ should not learn anything about the passwords
- No offline attacks
- $\mathcal{A}$ should only have one password guess per session

Existing Approaches from [DHP$^+$18]

- Garbled circuit based protocol for arbitrary distances
- Robust secret sharing based, for Hamming distance

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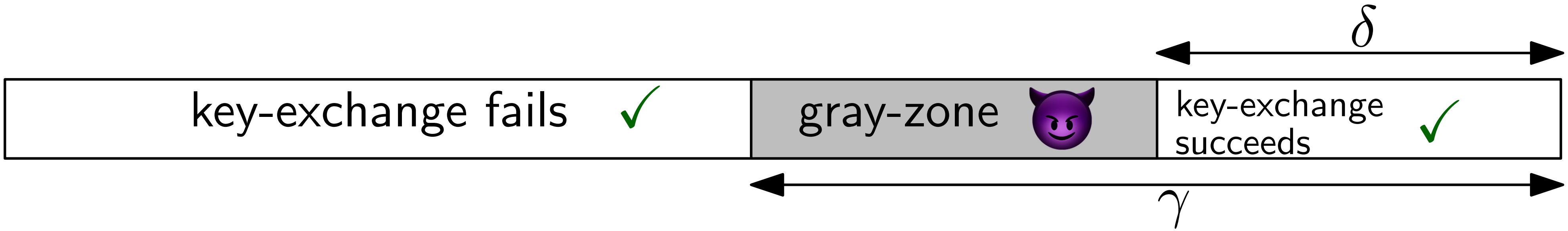

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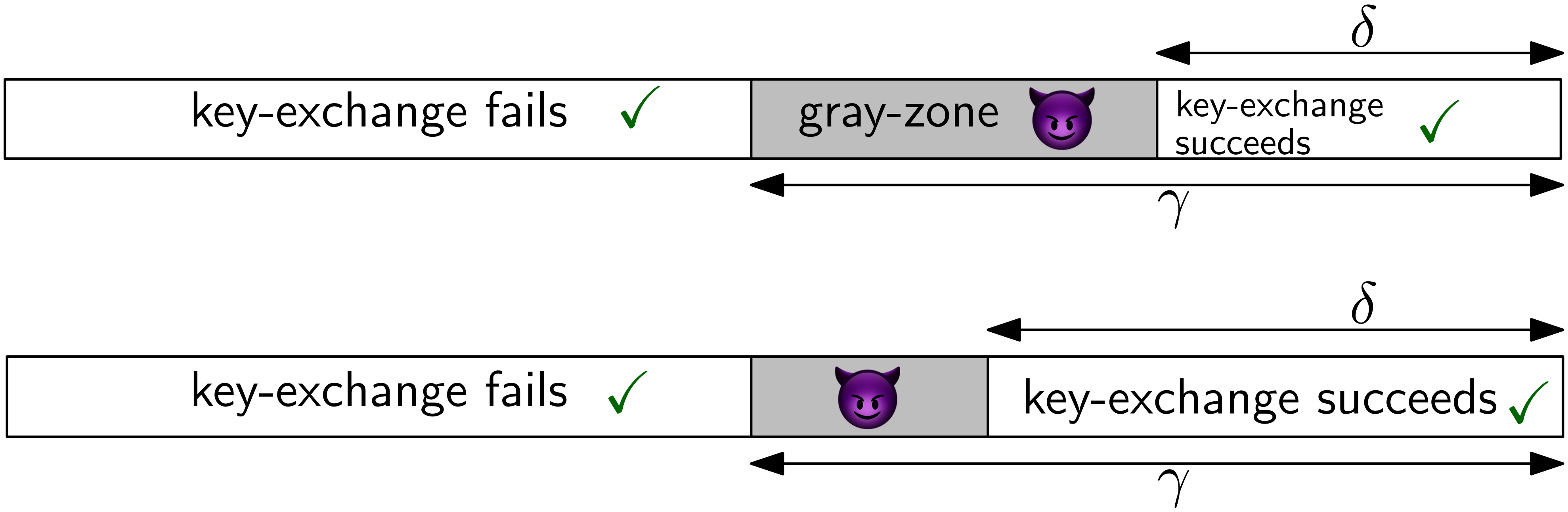

Intuition of the fPAKE-RSS protocol

$\mathsf{pw} = 01{\color{red}1}0$

$\mathsf{pw}' = 01{\color{red}0}0$

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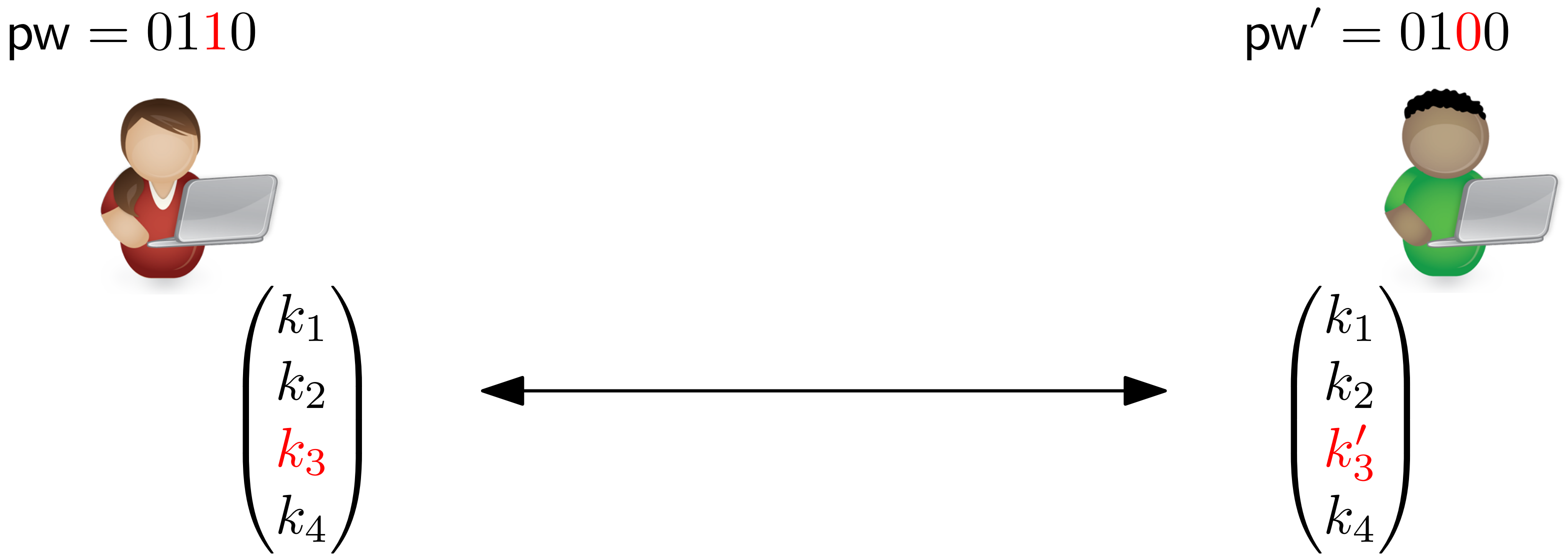
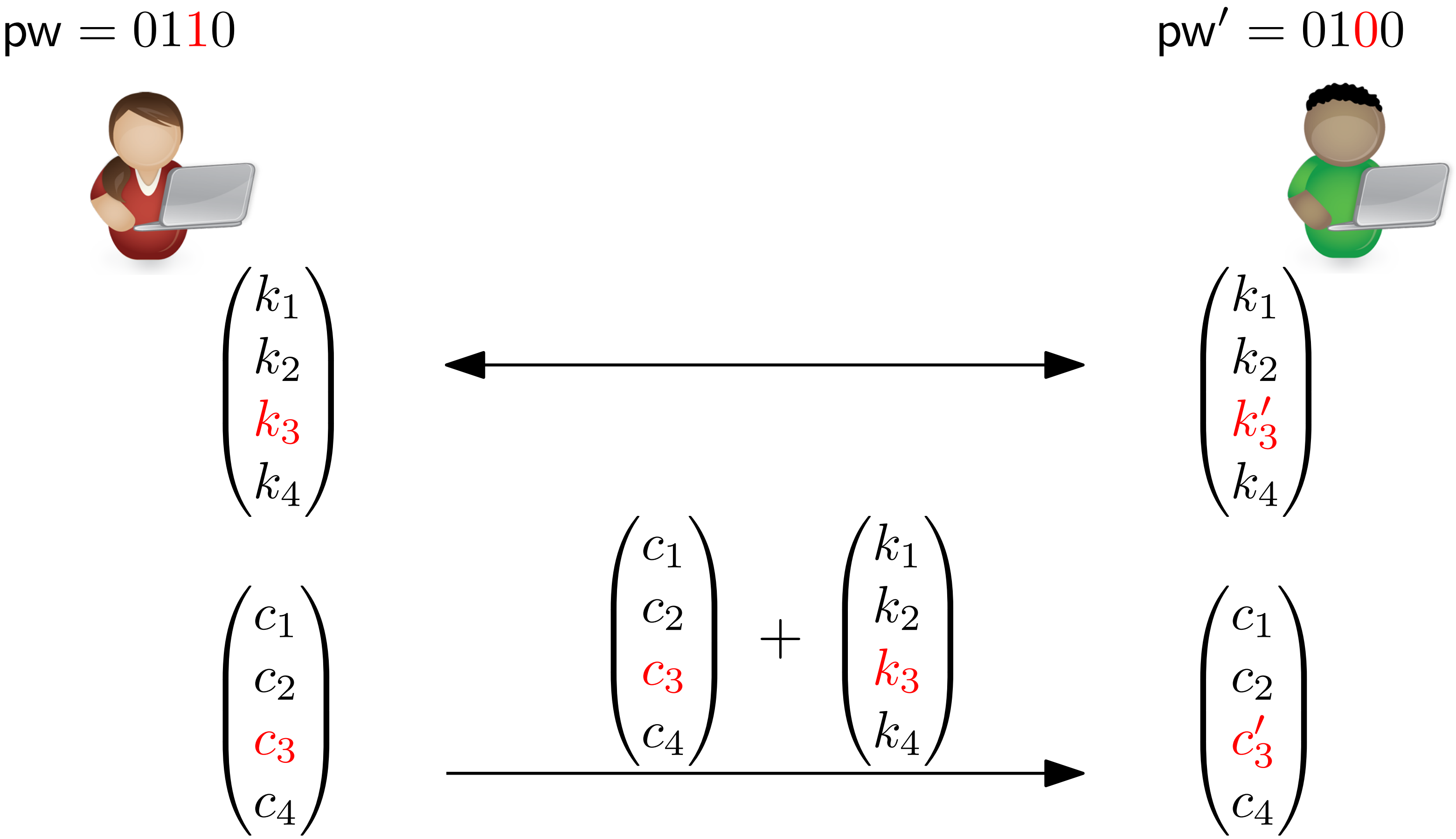


$\mathsf{pw} = 01\textcolor{red}{1}0$

$\mathsf{pw}' = 01\textcolor{red}{0}0$

$$\begin{pmatrix} k_1 \\ k_2 \\ \textcolor{red}{k_3} \\ k_4 \end{pmatrix} \longleftrightarrow \begin{pmatrix} k_1 \\ k_2 \\ \textcolor{red}{k'_3} \\ k_4 \end{pmatrix}$$

Intuition of the fPAKE-RSS protocol



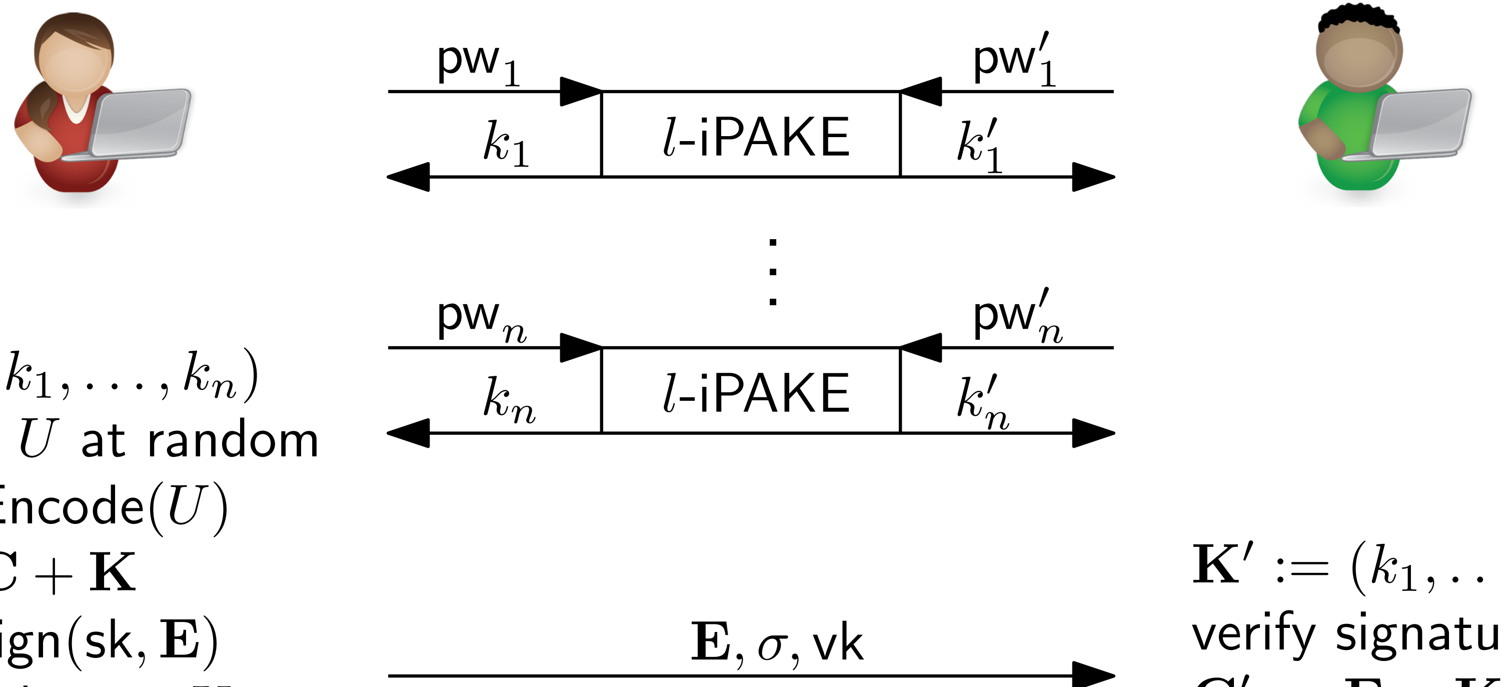
pw $= 0110$

pw$'$ $= 0100$

$$\begin{pmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{pmatrix} \longleftrightarrow \begin{pmatrix} k_1 \\ k_2 \\ k'_3 \\ k_4 \end{pmatrix}$$

$$\begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix} \xrightarrow{\begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix} + \begin{pmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{pmatrix}} \begin{pmatrix} c_1 \\ c_2 \\ c'_3 \\ c_4 \end{pmatrix}$$

Attack on the fPAKE-RSS protocol



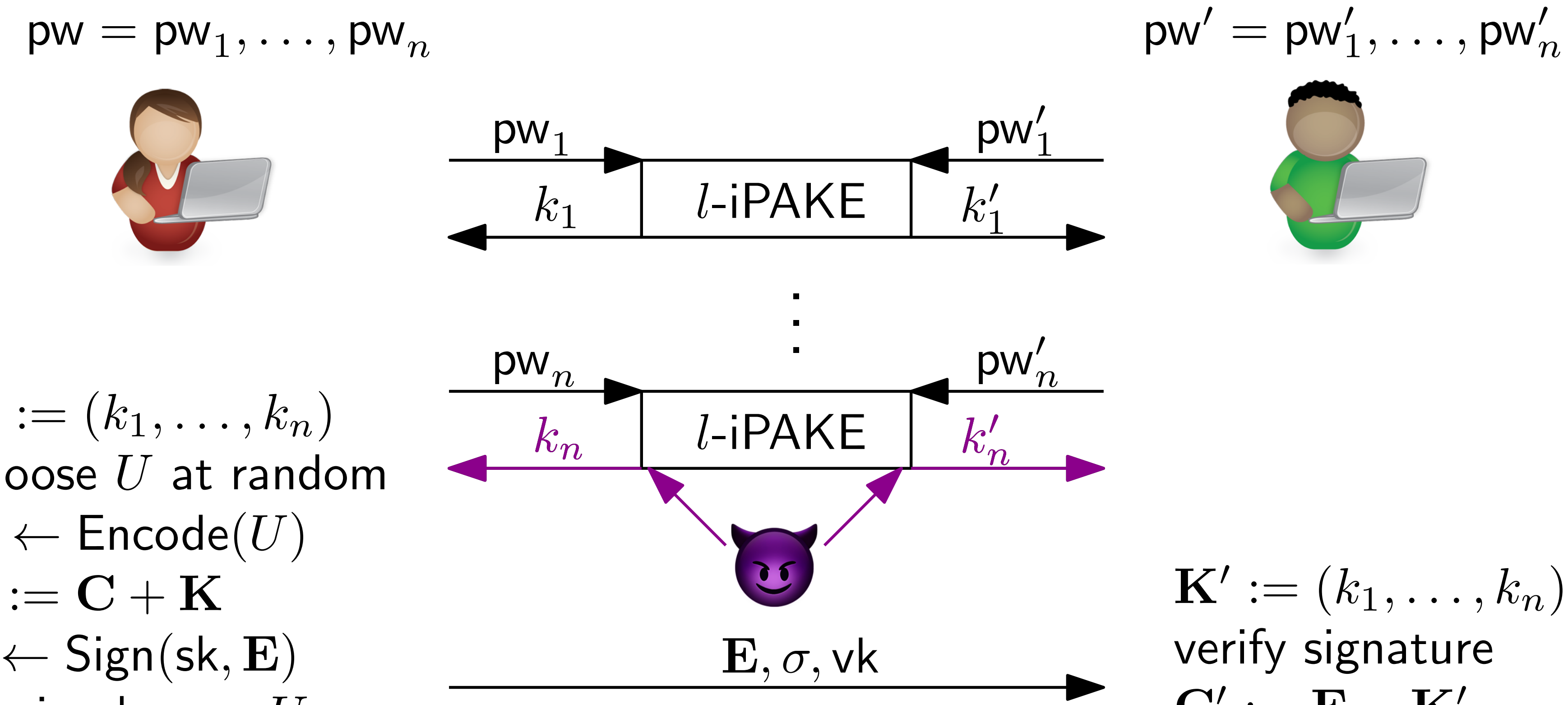
$\mathsf{pw} = \mathsf{pw}_1, \ldots, \mathsf{pw}_n$

$\mathsf{pw}' = \mathsf{pw}'_1, \ldots, \mathsf{pw}'_n$

$\mathbf{K} := (k_1, \ldots, k_n)$
choose U at random
$\mathbf{C} \leftarrow \mathsf{Encode}(U)$
$\mathbf{E} := \mathbf{C} + \mathbf{K}$
$\sigma \leftarrow \mathsf{Sign}(\mathsf{sk}, \mathbf{E})$
session-key $:= U$

$\mathbf{K}' := (k_1, \ldots, k_n)$
verify signature
$\mathbf{C}' := \mathbf{E} - \mathbf{K}'$
$U' \leftarrow \mathsf{Decode}(\mathbf{C}')$
session-key $:= U'$

Attack on the fPAKE-RSS protocol



$\text{pw} = \text{pw}_1, \ldots, \text{pw}_n$
$\text{pw}' = \text{pw}'_1, \ldots, \text{pw}'_n$
pw_1
pw'_1
k_1
l-iPAKE
k'_1
$\vdots$
pw_n
pw'_n
k_n
l-iPAKE
k'_n
$\mathbf{E}, \sigma, \text{vk}$

$\mathbf{K} := (k_1, \ldots, k_n)$
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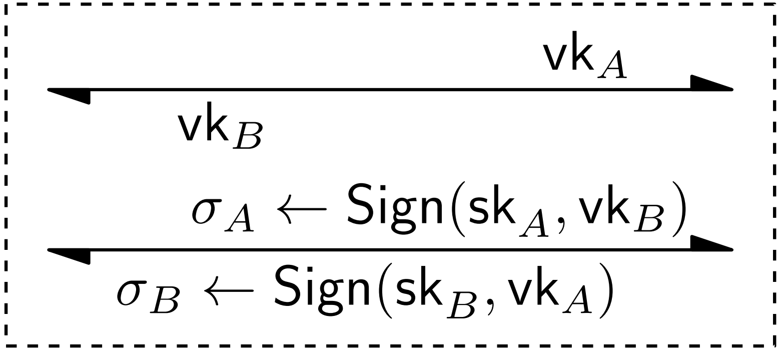
Summary of the Attack

- Requirements:
  - Both parties run the protocol
  - Both parties use the same password
  - The attacker actively interferes in the protocol
- Effect:
  - The attacker learns one bit of their choice of the password

Variations of the attack also work when the passwords are not identical

Overview of our fuzzy PAKE protocol

$\mathsf{pw} = \mathsf{pw}_1, \ldots, \mathsf{pw}_n$



vk_A

vk_B

$\sigma_A \leftarrow \mathsf{Sign}(\mathsf{sk}_A, \mathsf{vk}_B)$

$\sigma_B \leftarrow \mathsf{Sign}(\mathsf{sk}_B, \mathsf{vk}_A)$

$\mathsf{pw}' = \mathsf{pw}'_1, \ldots, \mathsf{pw}'_n$

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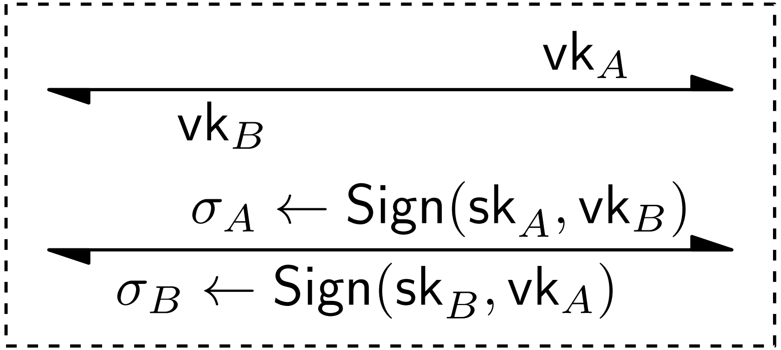


vk_A

vk_B

$\sigma_A \leftarrow \mathsf{Sign}(\mathsf{sk}_A, \mathsf{vk}_B)$

$\sigma_B \leftarrow \mathsf{Sign}(\mathsf{sk}_B, \mathsf{vk}_A)$

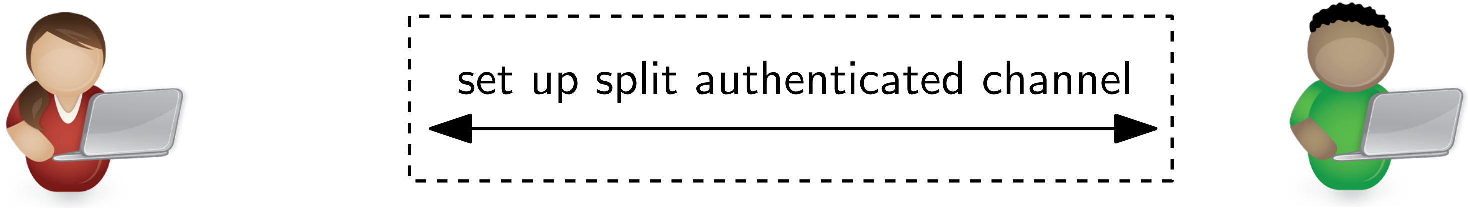
$\mathbf{C} \leftarrow \mathsf{Encode}(\mathsf{secret})$
$\mathbf{E} := \mathsf{blind}(\mathbf{C})$
$h := H(\mathbf{C})$

$\xrightarrow{\mathbf{E}, h}$

$\mathbf{C}' := \mathsf{unblind}(\mathbf{E})$
$L \leftarrow \mathsf{ListDecode}(\mathbf{C}')$
identify correct $\mathbf{C}$ via h
$\mathsf{secret} := \mathsf{Decode}(\mathbf{C})$

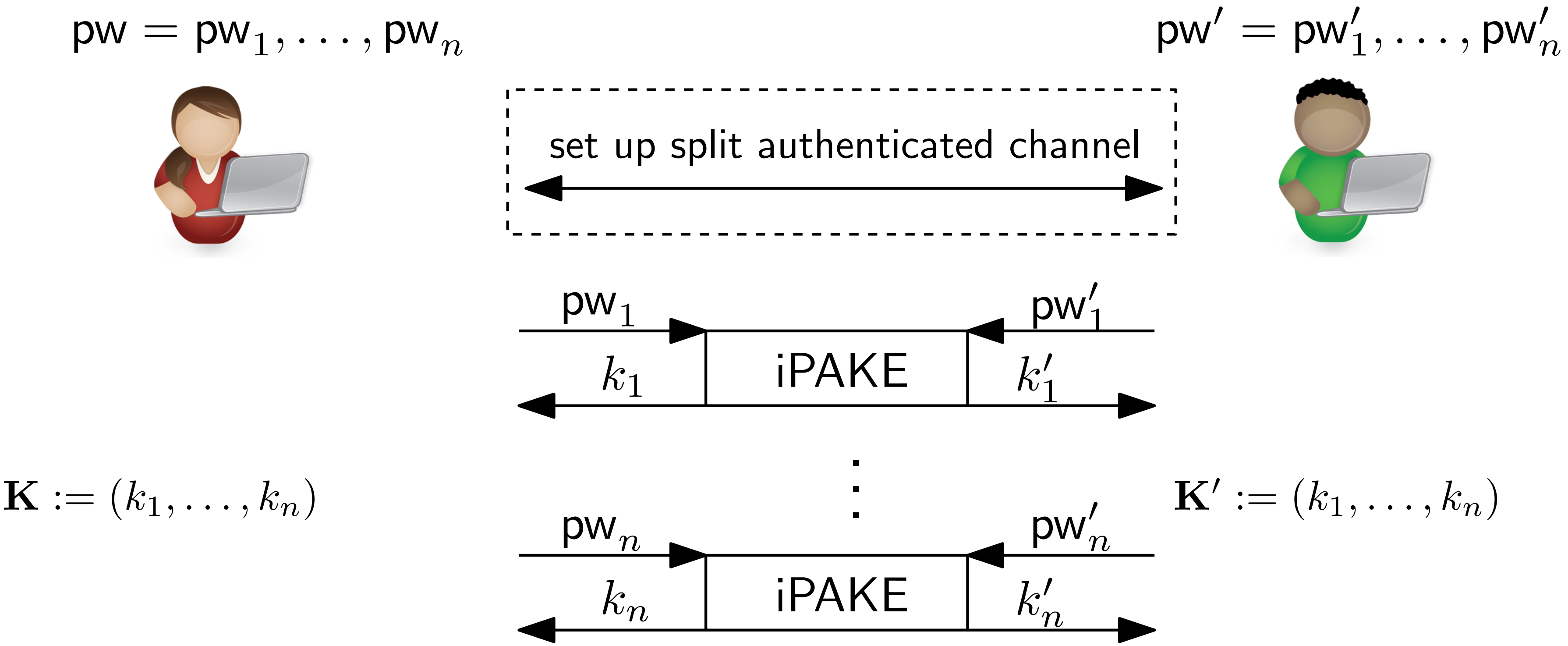
Our fuzzy PAKE protocol

$\mathsf{pw} = \mathsf{pw}_1, \ldots, \mathsf{pw}_n$ $\mathsf{pw}' = \mathsf{pw}'_1, \ldots, \mathsf{pw}'_n$



set up split authenticated channel

Our fuzzy PAKE protocol



$\mathsf{pw} = \mathsf{pw}_1, \ldots, \mathsf{pw}_n$

$\mathsf{pw}' = \mathsf{pw}'_1, \ldots, \mathsf{pw}'_n$

set up split authenticated channel

pw_1 iPAKE pw'_1

k_1 k'_1

$\vdots$

pw_n iPAKE pw'_n

k_n k'_n

$\mathbf{K} := (k_1, \ldots, k_n)$

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Our fuzzy PAKE protocol

$\mathsf{pw} = \mathsf{pw}_1, \ldots, \mathsf{pw}_n$

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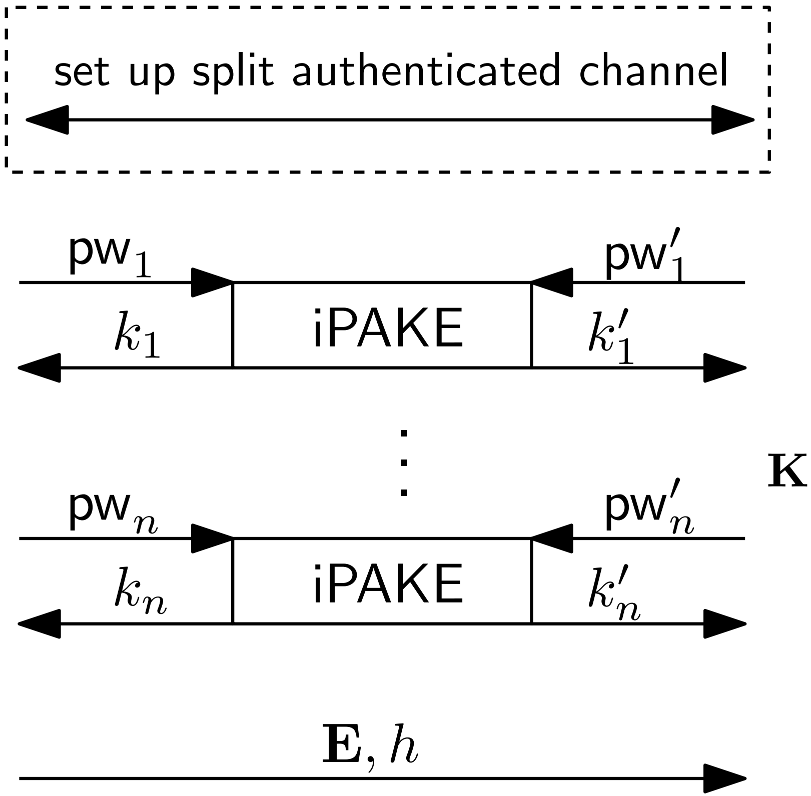

$\mathbf{K} := (k_1, \ldots, k_n)$

$\mathbf{K}' := (k_1, \ldots, k_n)$

$\mathbf{s} := (s_1, \ldots s_k) \xleftarrow{\$} \mathbb{F}_q^k$

$\mathbf{C} \leftarrow \mathsf{Encode}(\mathbf{s})$

$\mathbf{E} := \mathbf{C} \oplus \mathbf{K}$

$h := H_0(\mathbf{C})$

$\mathsf{session\text{-}key} := H_1(s_1)$

Our fuzzy PAKE protocol

$\mathsf{pw} = \mathsf{pw}_1, \ldots, \mathsf{pw}_n$

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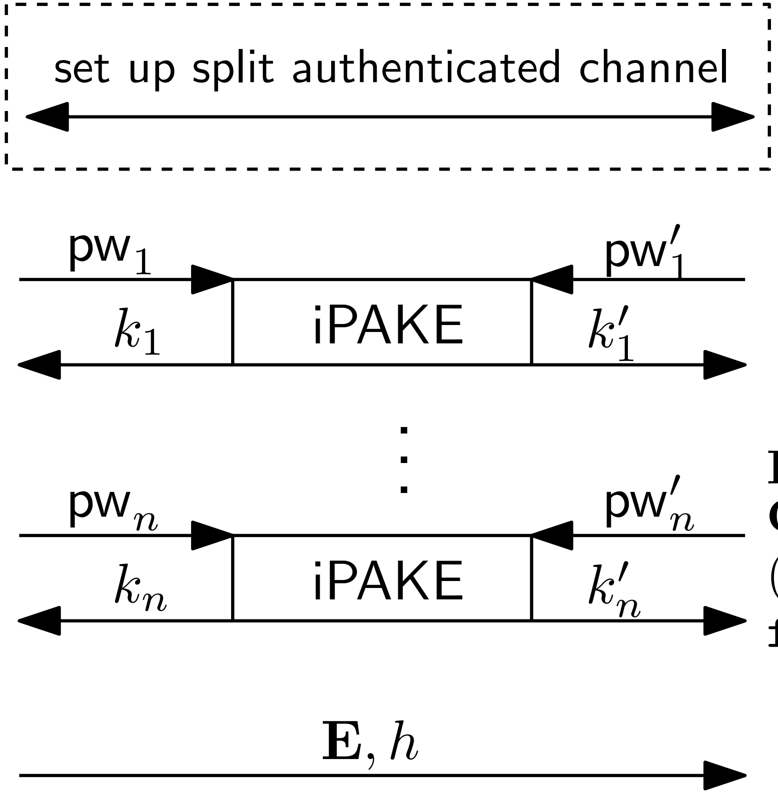

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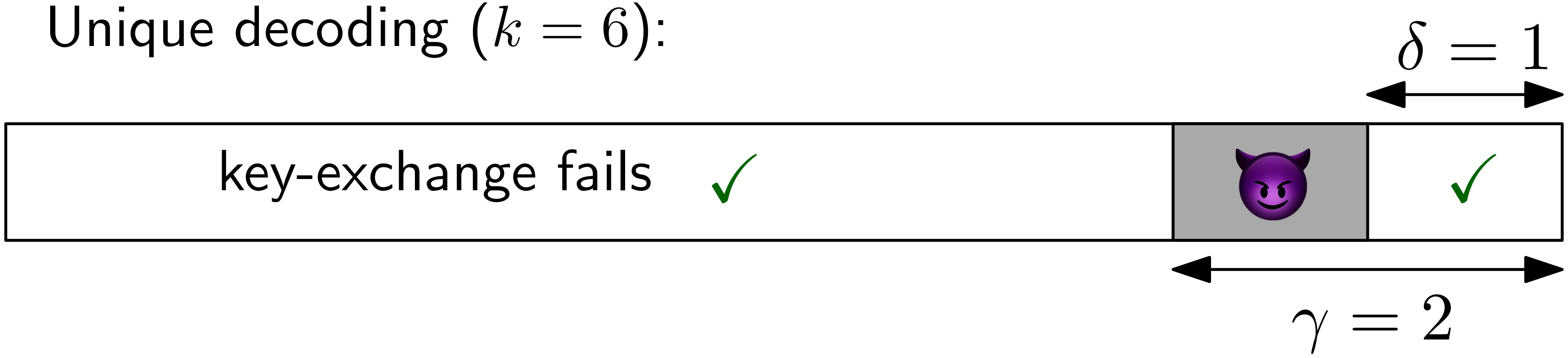
session-key $:= H_1(s_1)$
```

```
$\mathbf{K}' := (k_1, \ldots, k_n)$
$\mathbf{C}' := \mathbf{E} \oplus \mathbf{K}'$
$(\mathbf{C}^1, \ldots, \mathbf{C}^l) \leftarrow \mathsf{ListDecode}(\mathbf{C}')$
for $j \in \{1, \ldots, l\}$:
   if $H_0(\mathbf{C}^j) = h$:
      set $\mathbf{s}' := \mathsf{Decode}(\mathbf{C}^j)$
session-key $:= H_1(s'_1)$
```

Improvements through list decoding

Correcting 1 error of an 8 character password (with an $[n, k]$ Code)

Unique decoding ($k = 6$):



$\delta = 1$

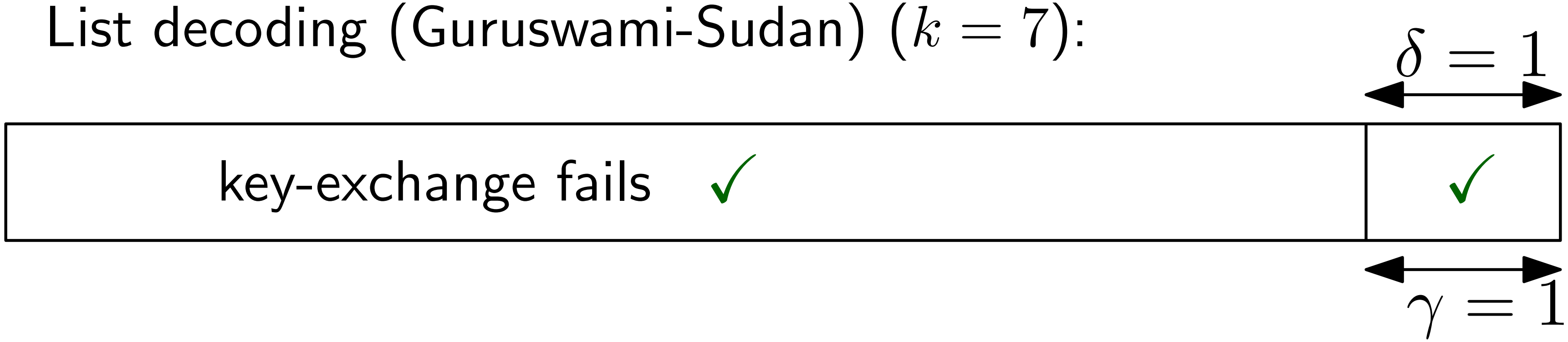
key-exchange fails ✓

😈

✓

$\gamma = 2$

List decoding (Guruswami-Sudan) ($k = 7$):



$\delta = 1$

key-exchange fails ✓

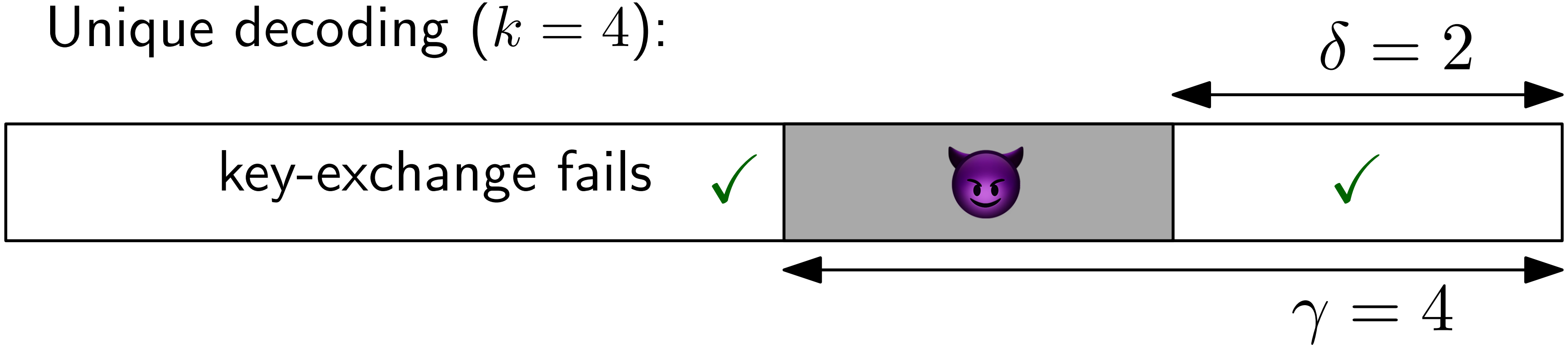
✓

$\gamma = 1$

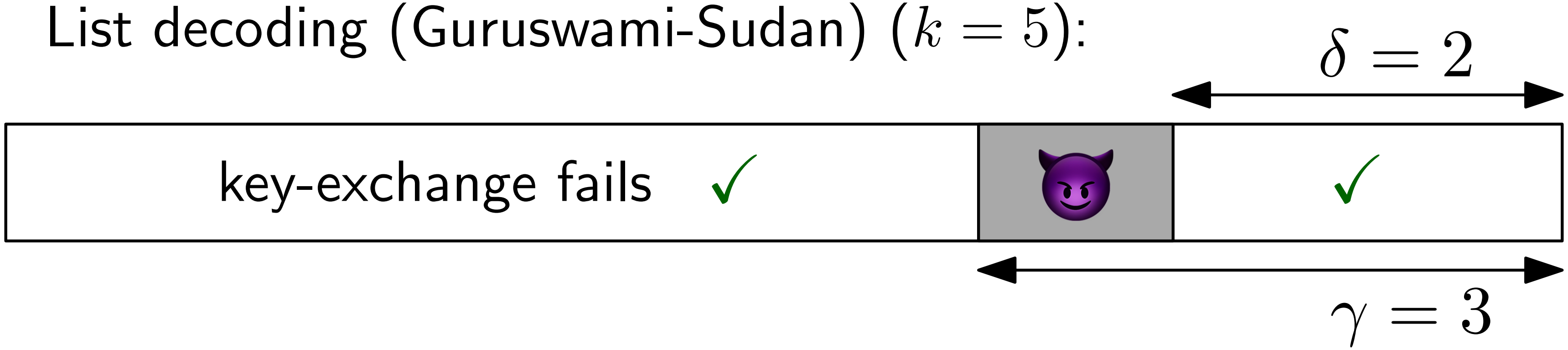
Improvements through list decoding

Correcting 2 error of an 8 character password (with an $[n, k]$ Code)

Unique decoding ($k = 4$):



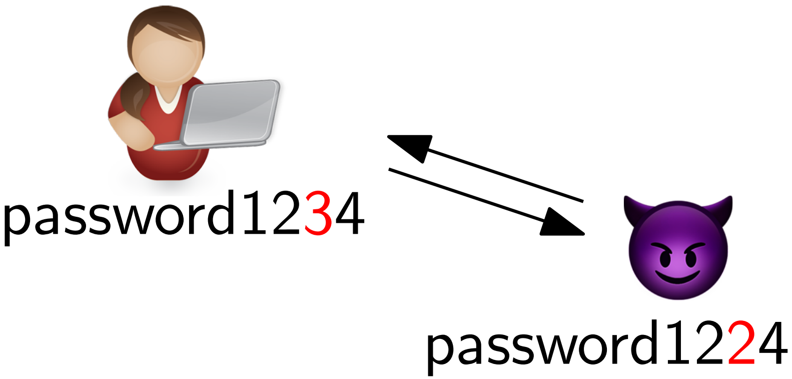
List decoding (Guruswami-Sudan) ($k = 5$):



Summary

- Attack on existing ECC-based fPAKE protocol
- New ECC-based fPAKE protocol
  - fix weakness of old protocol
  - better decoding via list decoding and hint
  - security analysis in UC framework

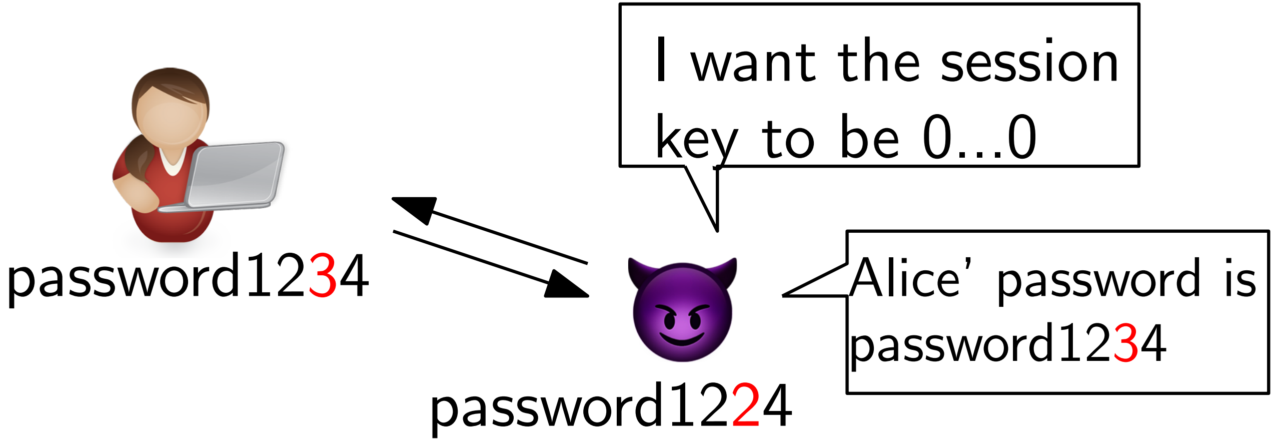
Outlook and PQ-Instantiations



password1234

password1224

Outlook and PQ-Instantiations



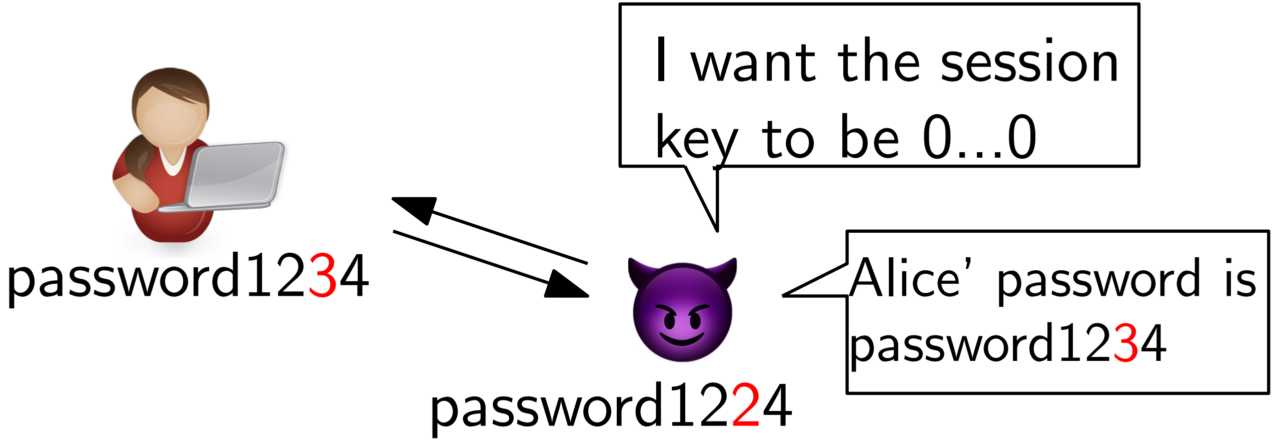
password1234

I want the session key to be 0...0

password1224

Alice' password is password1234

Outlook and PQ-Instantiations



I want the session key to be 0...0

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Alice’ password is password1234

password1224

- Instantiation of underlying PAKE needs to be *implicit-only*
    - CAKE protocol of [BCP$^+$23] is a promising candidate

Questions

Paper at `ia.cr/2023/1415`