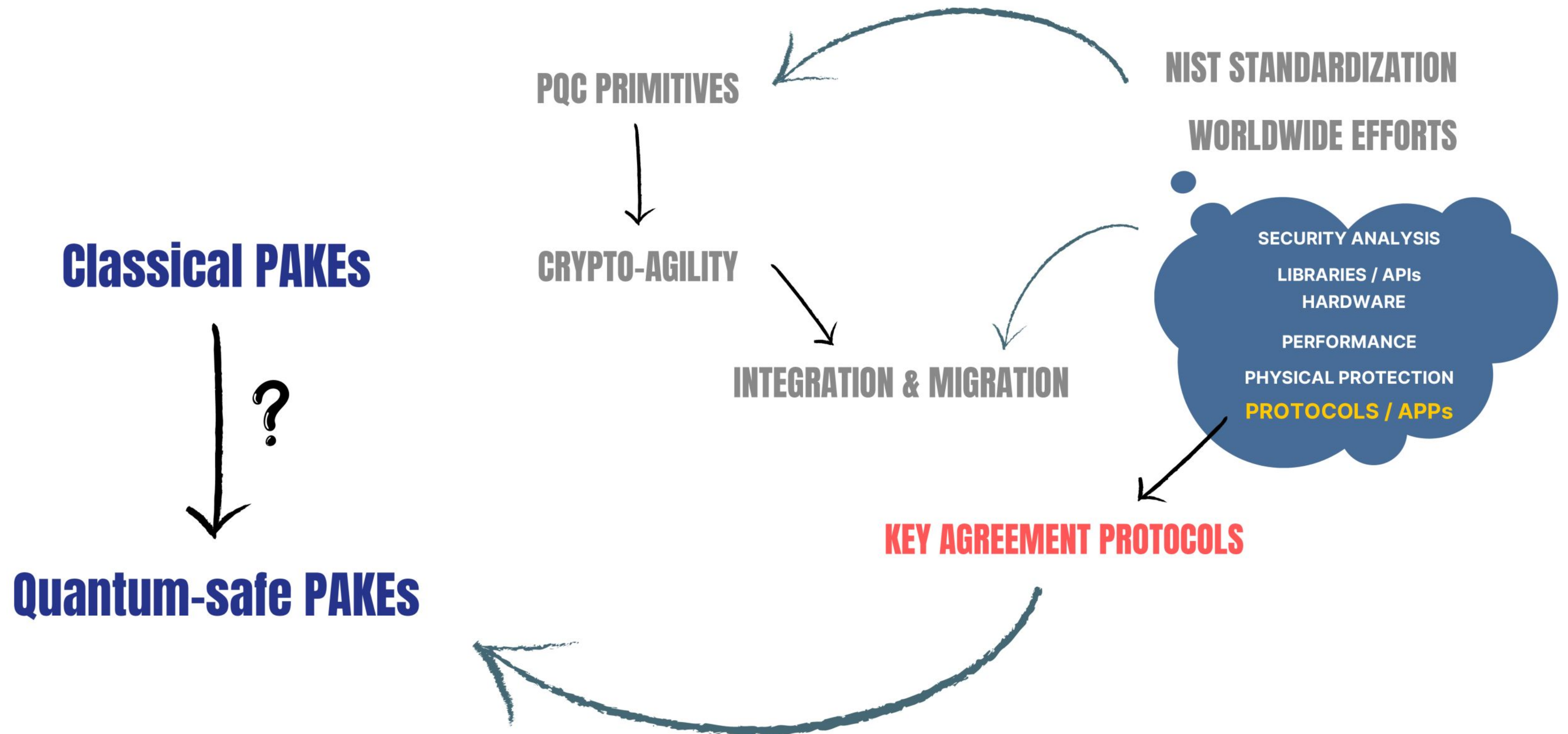


PQC PAKES

AN OVERVIEW

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PQC Workshop - Darmstadt - 2025

MOTIVATION



KEY EXCHANGE AND PAKE

KEX

Ephemeral Keys

Key Agreement

Symmetric Key

AKE

Static Keys

Authentication

Key Agreement

PAKE

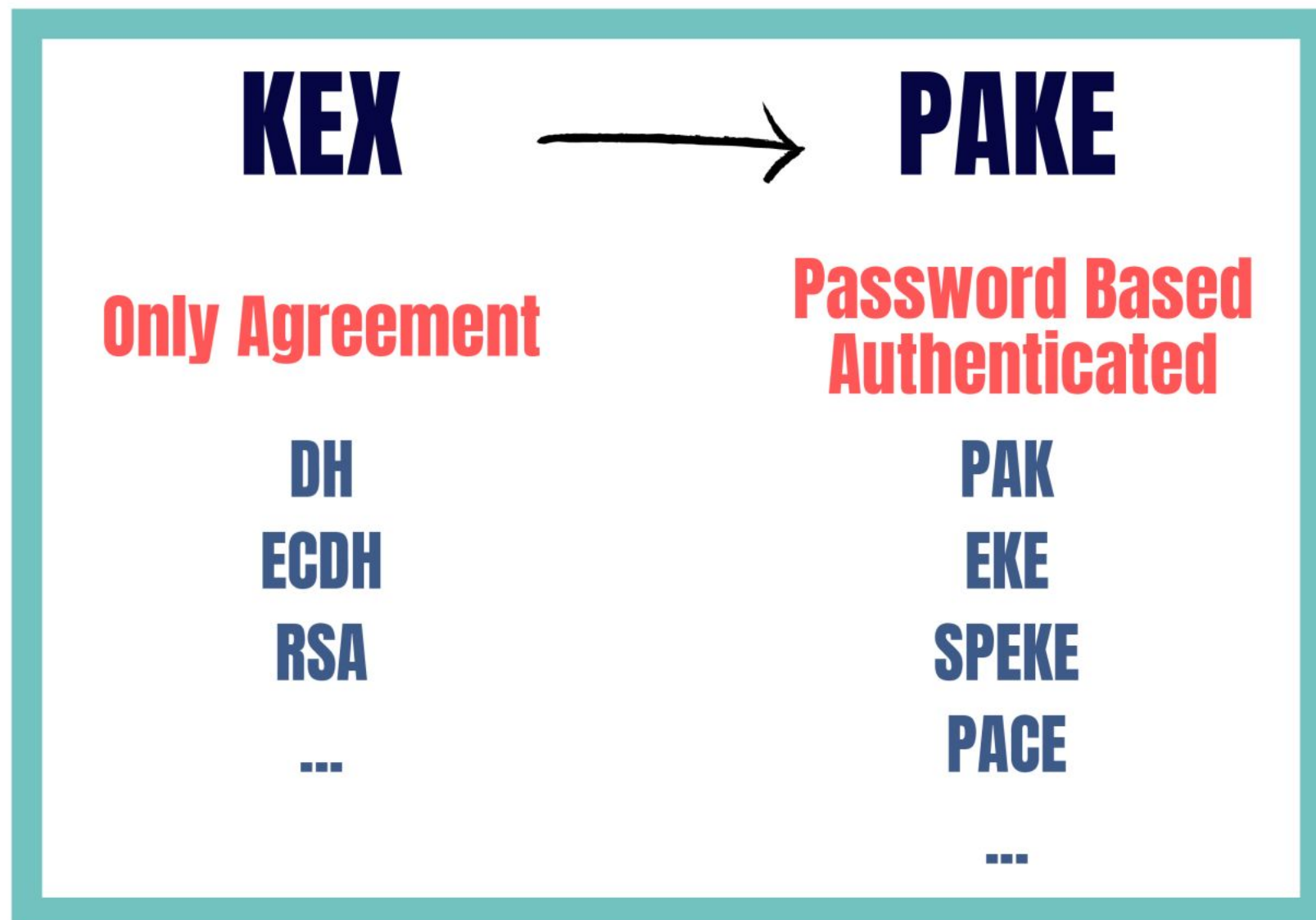
Low Entropy Password

Authenticated Public Key

Key Agreement

FROM KEX TO PAKE

- Many designs use a password to authenticate a DH or RSA key agreement

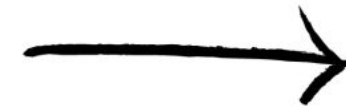


FROM CLASSICAL TO PQC

- Replace DH in PACE by a PQC KEX?



PQC KEX



PQC PAKE

Only Agreement

**Password Based
Authenticated**

**Peikert-KEX
DING-KEX**

...



PQC KEY AGREEMENT

- DH and SIDH are commutative, contributive and non-interactive
- LWE is semi-commutative (error reconciliation)
- Early LWE schemes rely on signaling and are thus interactive

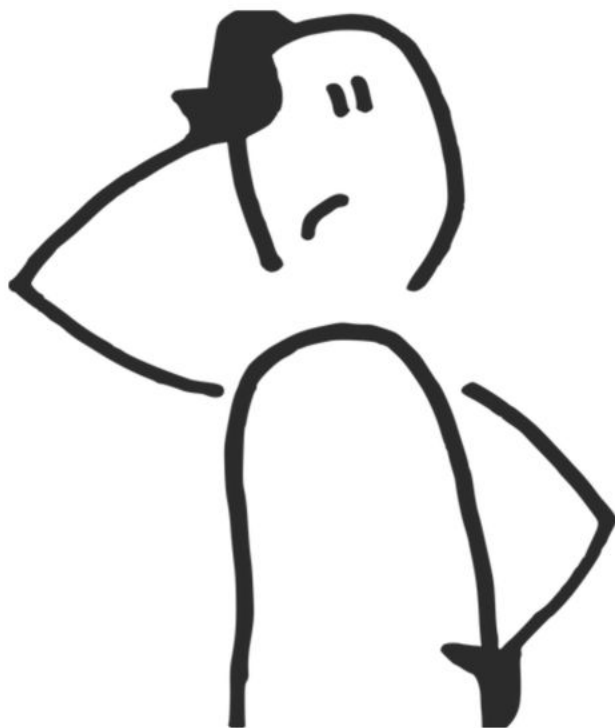
Classical PAKEs

???

Direct PQC PAKEs

Non-NIST
Lattices & Isogenies

	(EC)DH	SIDH	LWE
G	$g \in \mathbb{G}$	$(P_i, Q_i)_i$	$\mathbf{A} \in R_q^{k \times k}$
a	$a \in \mathbb{G} $	ϕ_A	$(\mathbf{s}_a, \mathbf{e}_a)$ short
A	g^a	$E_A, \phi_A(E_2)$	$\mathbf{s}_a^t \cdot \mathbf{A} + \mathbf{e}_a^t$
B	g^b	$E_B, \phi_B(E_3)$	$\mathbf{A} \cdot \mathbf{s}_b + \mathbf{e}_b$
h	-	-	Yes
Static?	Yes	No	No



DING-RLWE PAK

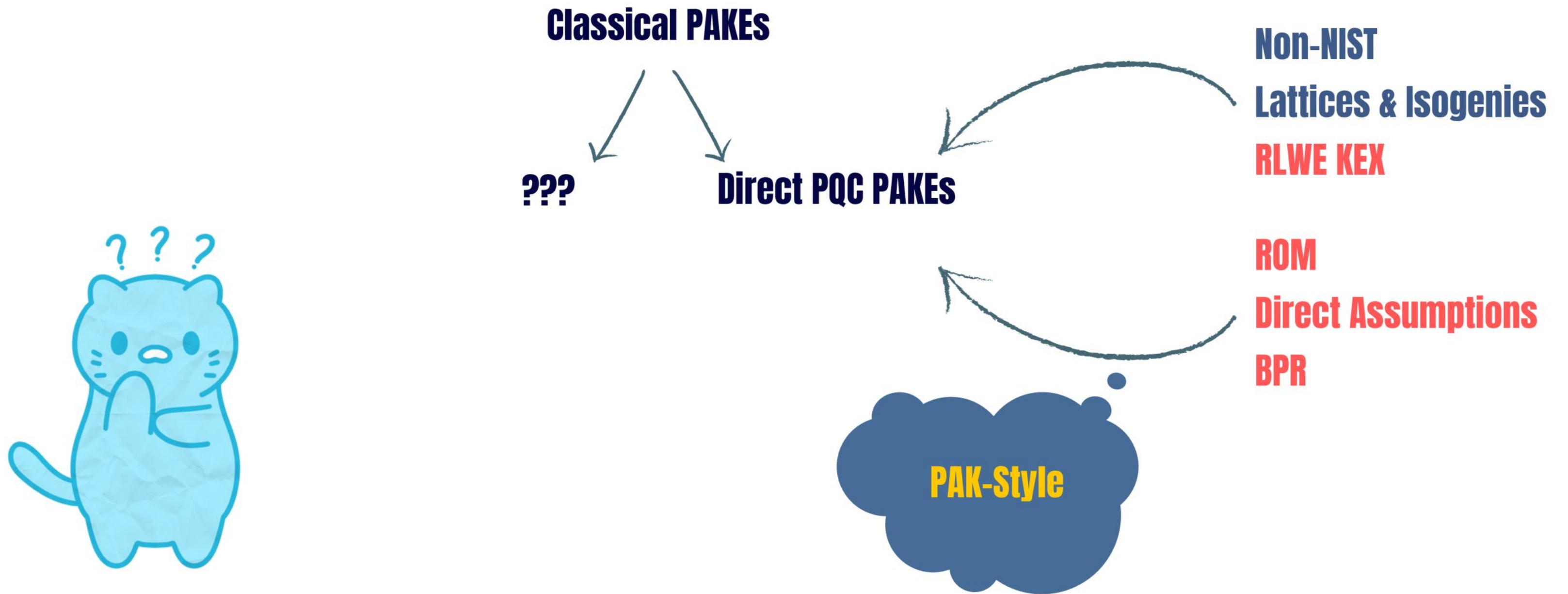
- Based on Mackenzie's PAK
- Uses Ding's RLWE crypto-system
- Requires signaling for reconciliation



$$(\mathbf{s}_a^t \cdot \mathbf{A} + \mathbf{e}_a^t) \mathbf{s}_b \approx \mathbf{s}_a^t (\mathbf{A} \cdot \mathbf{s}_b + \mathbf{e}_b)$$

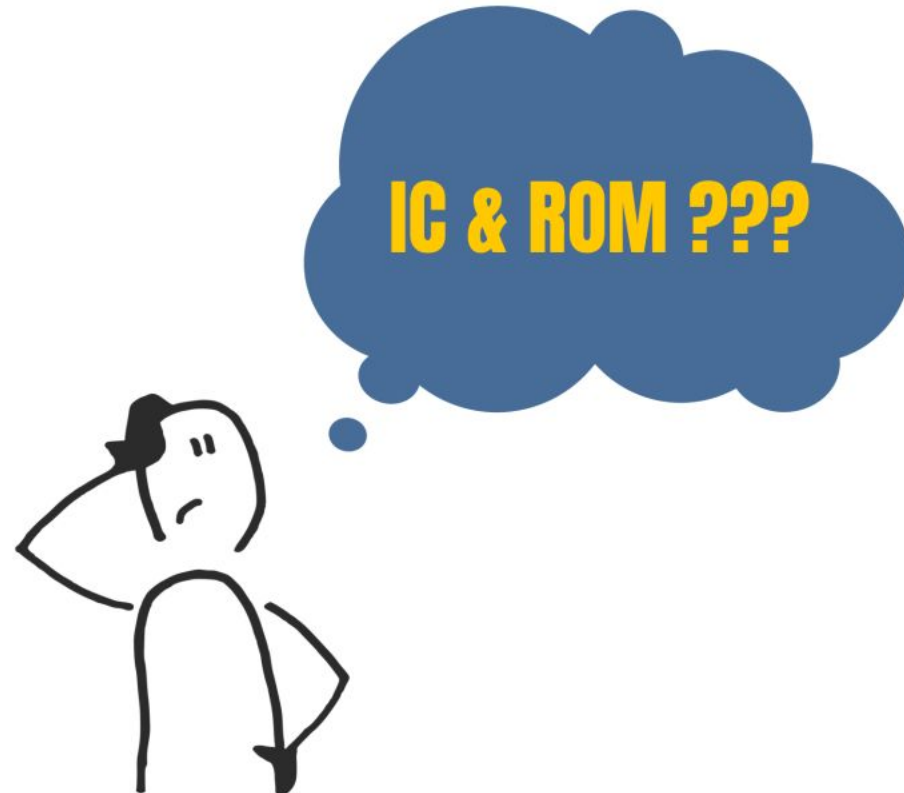
Alice		Bob
Password π		Password π
Mapping Protocol		
generate $\mathbf{s}_A, \mathbf{e}_A \in \mathbb{R}_q[\chi]$ $\mathbf{p}_A = \mathbf{M}\mathbf{s}_A + 2\mathbf{e}_A$ $\mathbf{v}_A = \mathbf{p}_A + H(\pi)$	$\left(\begin{array}{l} \leftarrow \mathbf{p}_B \\ \rightarrow \mathbf{v}_A \end{array} \right)$	generate $\mathbf{s}_B, \mathbf{e}_B \in \mathbb{R}_q[\chi]$ $\mathbf{p}_B = \mathbf{M}\mathbf{s}_B + 2\mathbf{e}_B$ $\mathbf{p}_A = \mathbf{v}_A - H(\pi)$
authentication token $sid = (A, B, \mathbf{v}_A, \mathbf{p}_B, \sigma, -H(\pi))$		
$K_A = \mathbf{p}_B^T \mathbf{s}_A$ $= \mathbf{M}^T \mathbf{s}_B^T \mathbf{s}_A + 2\mathbf{e}_B^T \mathbf{s}_A$	$\left(\begin{array}{l} \leftarrow w_B \\ \rightarrow T_A \\ \leftarrow T_B \end{array} \right)$	$K_B = \mathbf{p}_A^T \mathbf{s}_B$ $= \mathbf{s}_A^T \mathbf{M}^T \mathbf{s}_B + 2\mathbf{e}_A^T \mathbf{s}_B$ $w_B = CHA(K_B)$ $\sigma = MOD_2(K_B, w_B)$
$\sigma = MOD_2(K_A, w_B)$ $T_A = H_A(sid)$		$T_B = H_B(sid)$ abort if $T_A \neq H_A(sid)$
abort if $T_B \neq H_B(sid)$	Establish key	
$K_{AB} = KDF(sid)$		$K_{AB} = KDF(sid)$

OVERVIEW SO FAR...



(O)EKE & (O)CAKE

- Generic design based on (O)EKE [8]
- Do not establish a new generator
- Authenticate the public key
- Encrypt public key instead of modifying it
- **Relies on the ROM and IC model**

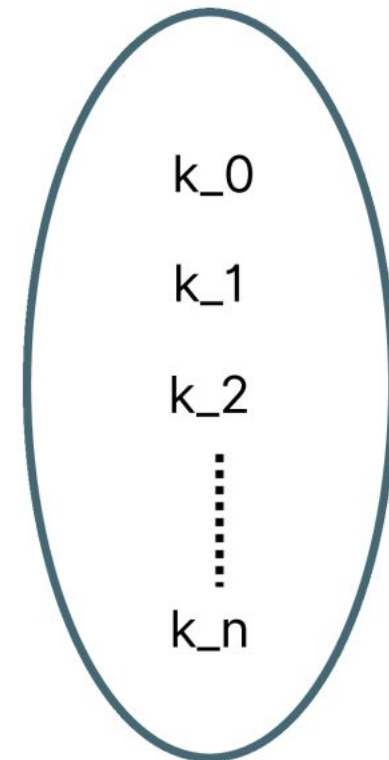


Alice	Bob
<i>Password</i> π	<i>Password</i> π'
$K_\pi = \mathcal{KDF}(\pi)$ $sk_a, pk_a \xleftarrow{\$} \text{KeyGen}$ $apk_a \xleftarrow{\$} C_{K_\pi}(pk)$	$K_{\pi'} = \mathcal{KDF}(\pi')$
$\xrightarrow{apk_a}$	
	$pk'_a = \mathcal{C}_{K_{\pi'}}^{-1}(apk_a)$
	$(c_b, \overline{K}) = \text{Encap}(pk'_a)$
$\xleftarrow{c_b}$	
$\overline{K}^* = \text{Decap}(sk_a, c_b)$ $K = \mathcal{KDF}(\overline{K}^*)$	$K = \mathcal{KDF}(\overline{K})$

ROM & IC

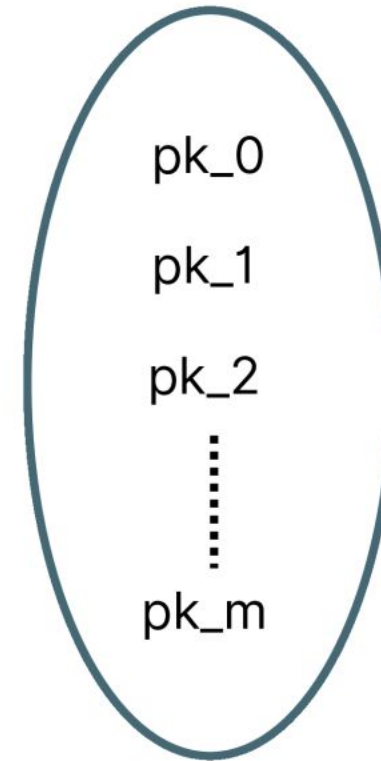
- Deterministic mapping of inputs to outputs
- Deterministic mapping of permutations in both directions

$\{0-9\}^{\text{pi}}$

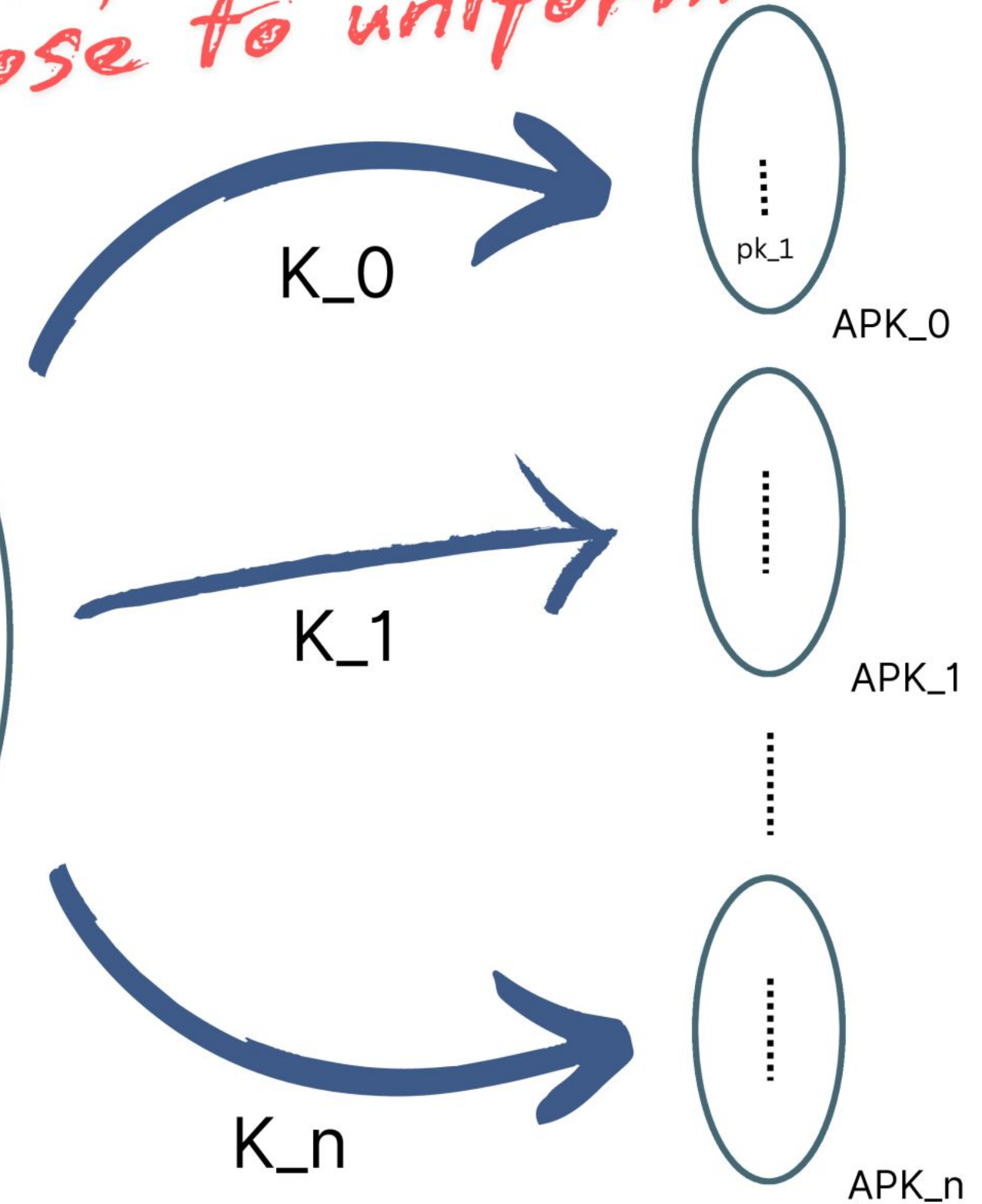


K

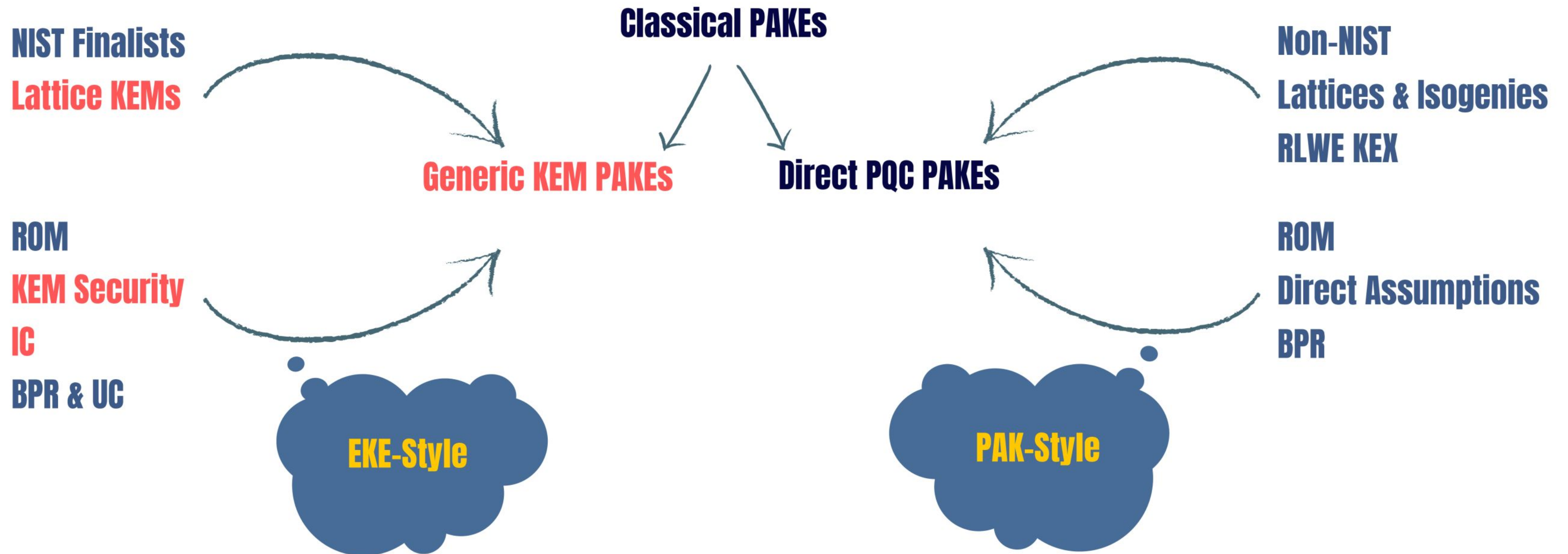
Kyber keys are only computationally indistinguishable from uniform, but not statistically close to uniform



PK



OVERVIEW SO FAR...



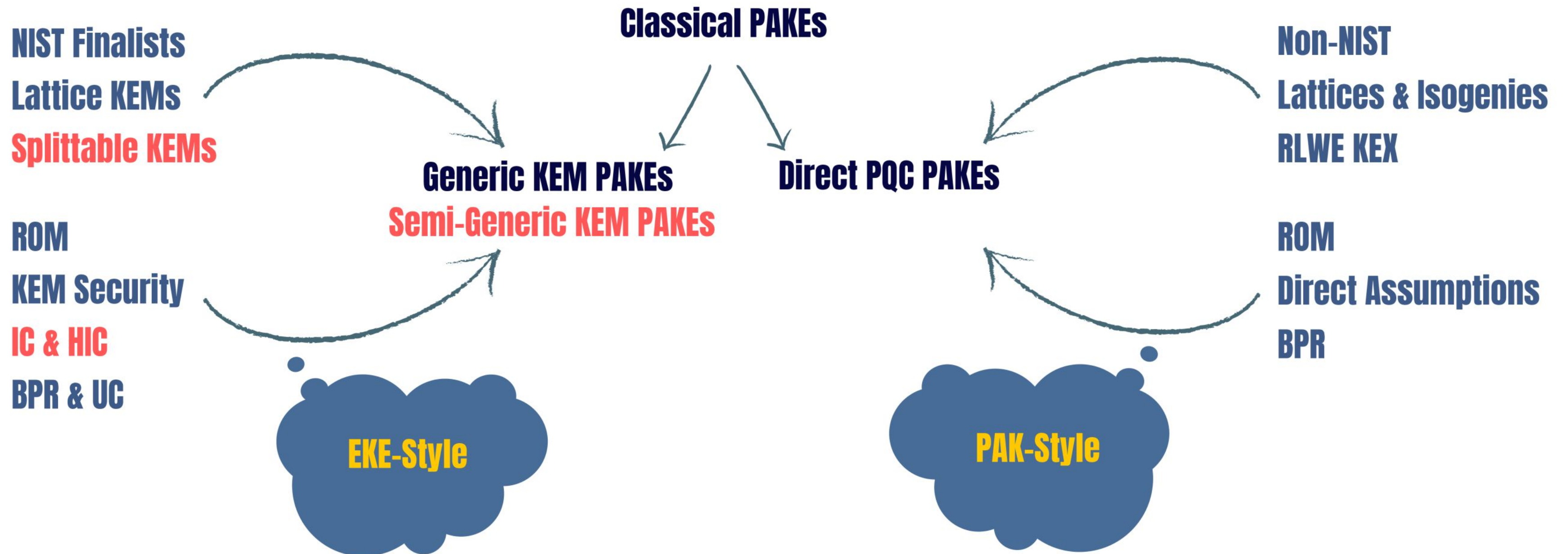
HALF IC & SPLITTABLE LWE KEYS

- Randomized Half IC on Groups by Dos Santos, Gu and Jarecki (EC'23)
- **C'est très CHIC** by Arriaga, Barbosa, Jarecki and Skrobot (AC'24)

Split a key and use the uniform bit string part to achieve a domain extension!

- LWE, RLWE and MLWE public keys of the form $(A, \langle b = As + e \rangle)$
- A is sampled from a uniform random bit string of fixed length (call it ρ !)
- Use lattice base seed for an IC encryption instead of a structured public key

OVERVIEW SO FAR...



ADDING OTHER COMPONENTS

NIZKs

Zero Knowledge Proofs



Public key as CRS



KOY-GL or PAK PAKEs

SPHFs

Hashing



Complicated stuff...



KOY-GL or PAK PAKEs

CRS

Pre-Registration

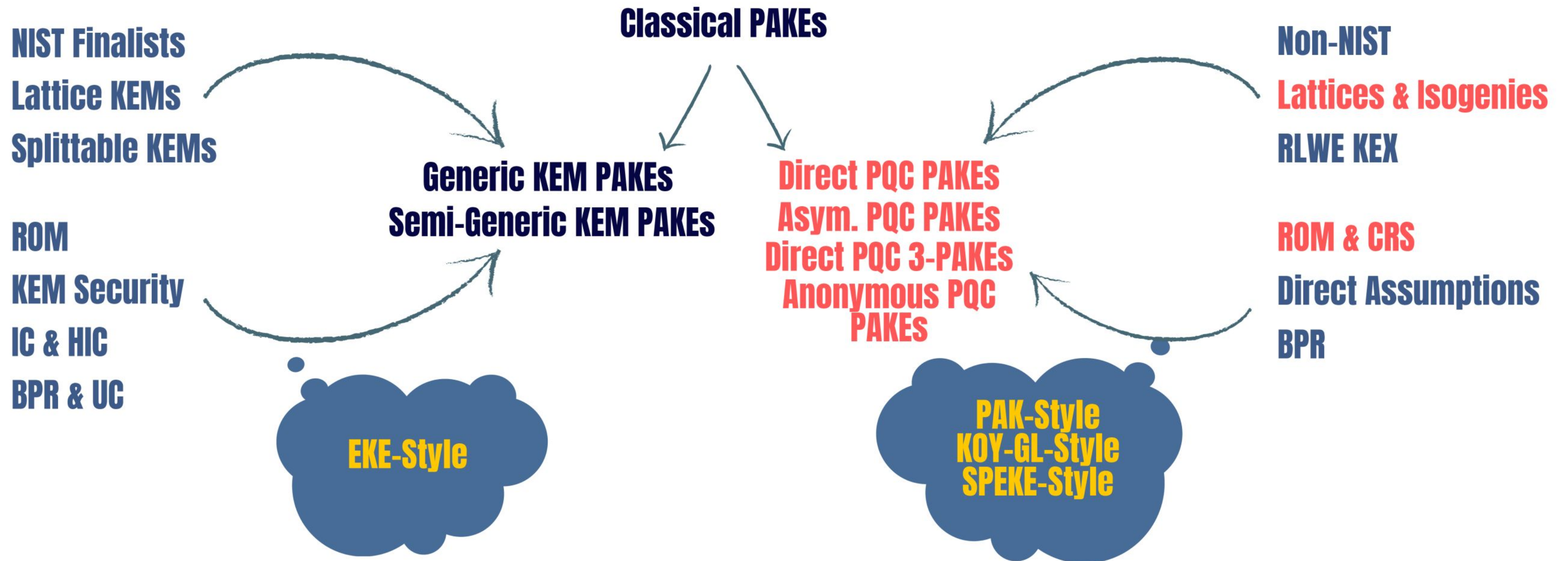


Static Public Key



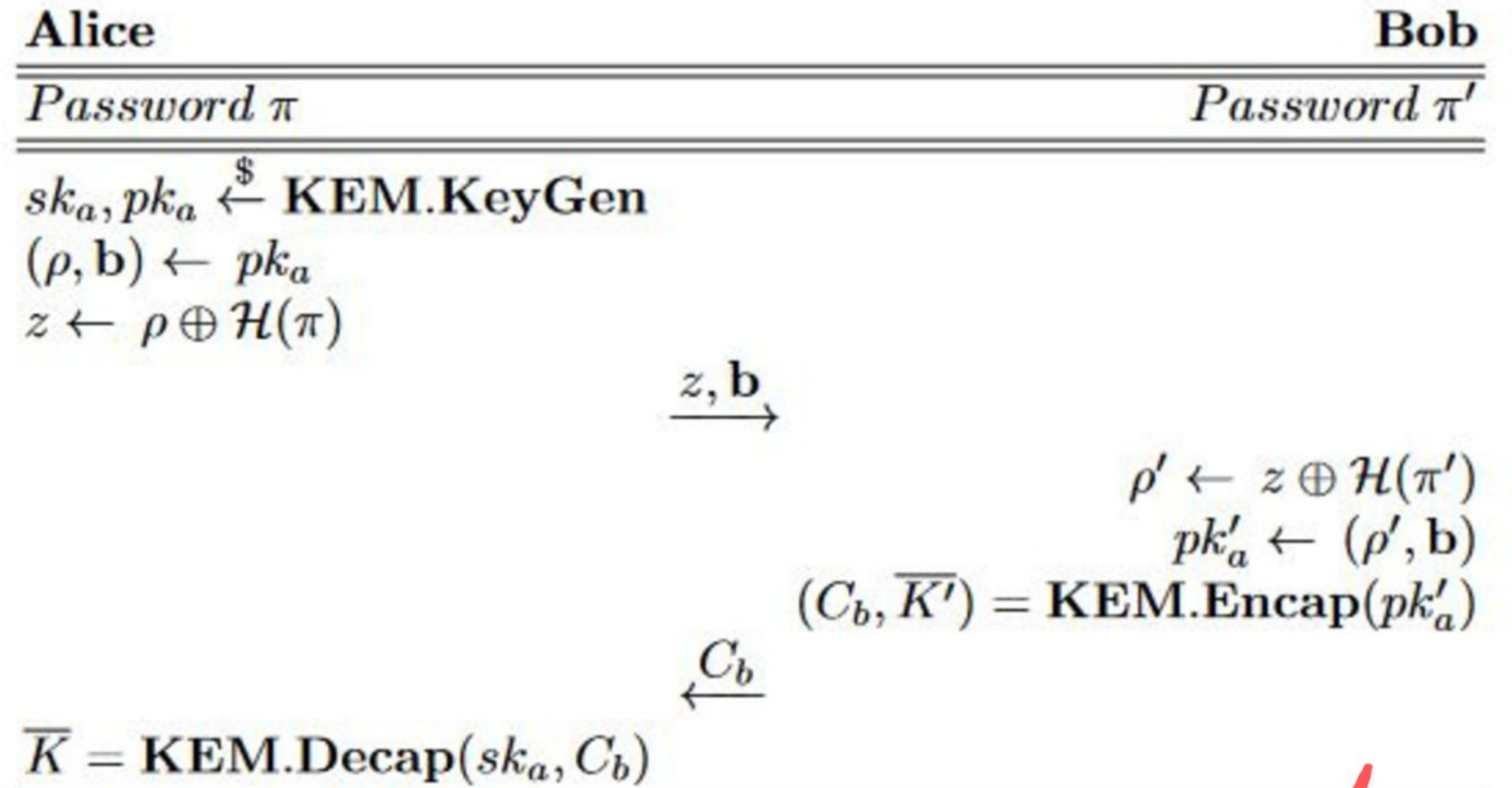
KOY-GL or PAK PAKEs

OVERVIEW SO FAR...



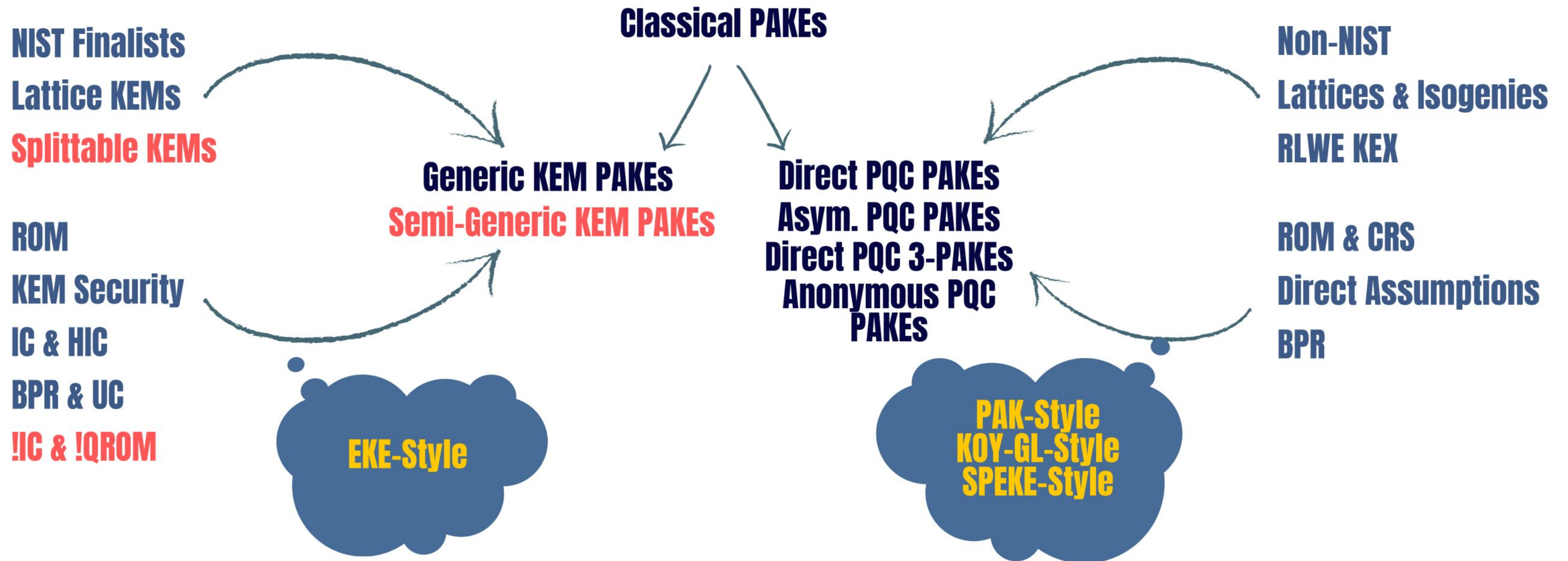
NICE-PAKE

- Splittable KEMs from LWE, RLWE and MLWE
- Inspired by CHIC
- Authenticate sampling seed of lattice base
- Simple XOR of hash output from password
- **No IC or QROM involved in proof!**



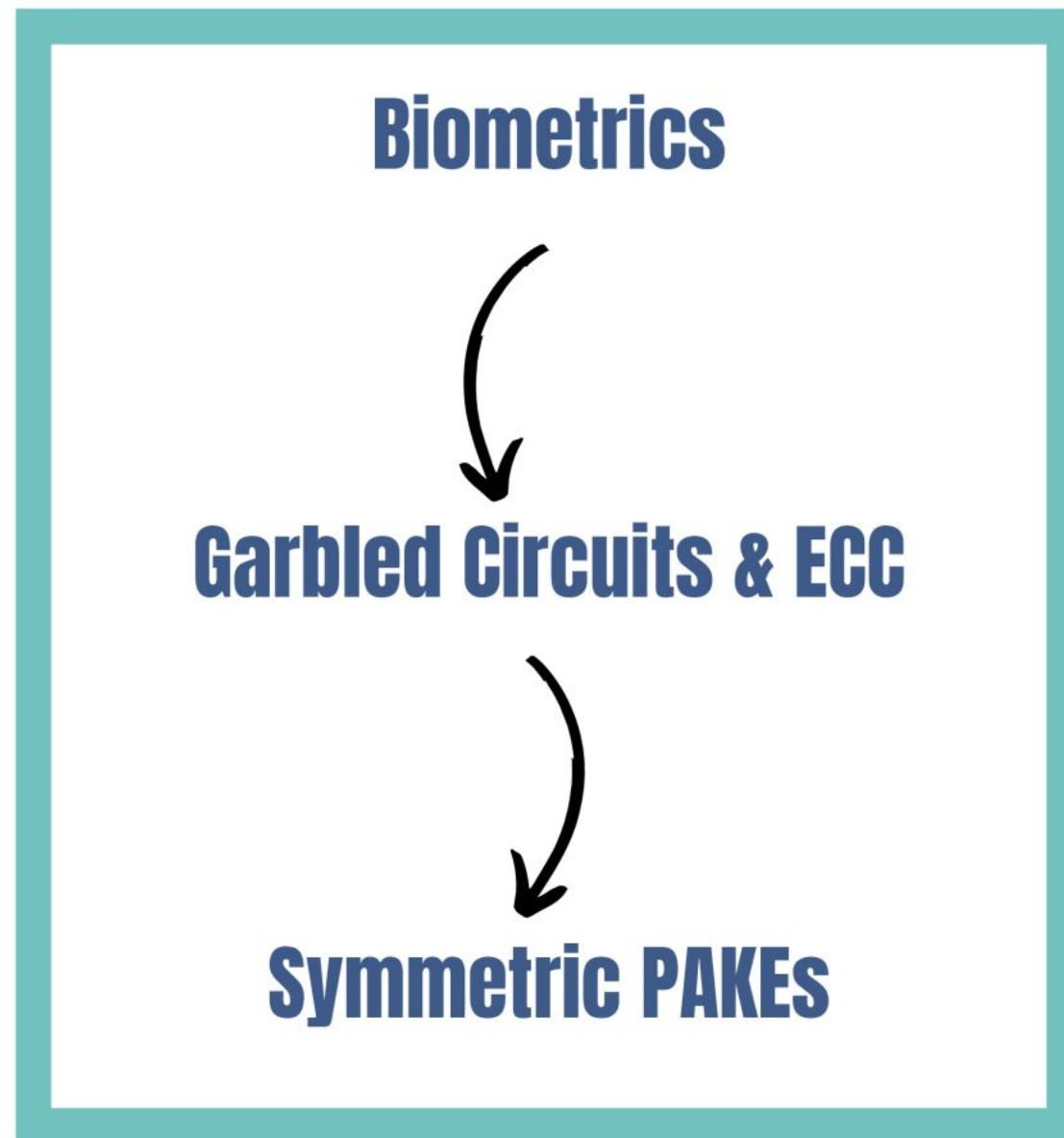
Relies on specific lattice assumptions and
requires additional KEM security preproperties!

OVERVIEW SO FAR...

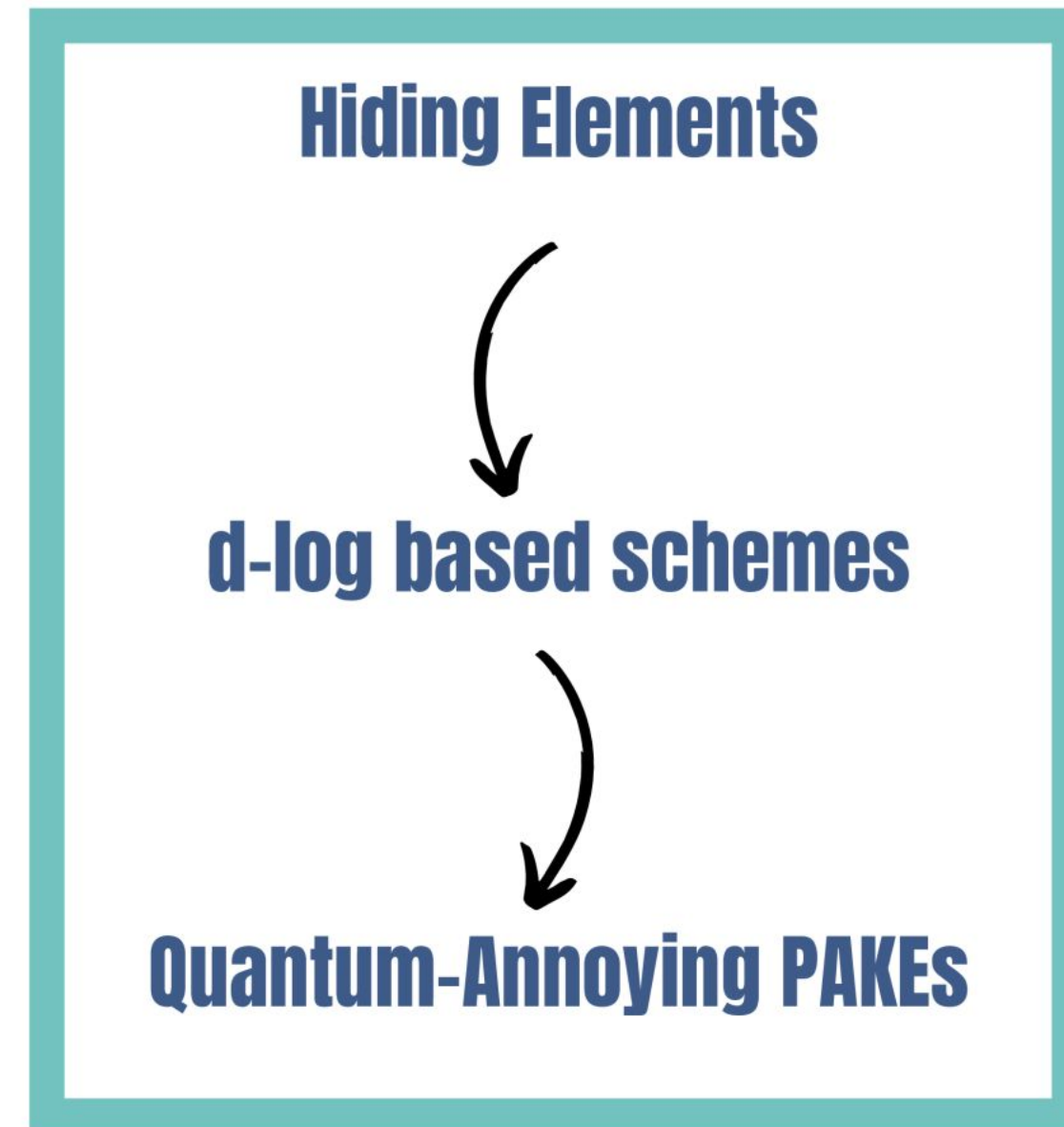


HONORABLE MENTIONS

Fuzzy



Annoying



OVERVIEW SO FAR...

- Only one PQC PAKE in the QROM
- Only four in UC
- Almost all from lattices
- Almost all based on DING-RLWE PAK



						Peer-Reviewed Quantum-Safe Generic	2-Party Benchmarks	3-Party Benchmarks	Recomm. Param. Provide Impl.
Balanced									
C1	Terada, Yoneyama [TY19] ((C)SIDH-EKE)	2019	IC, ROM	SIDH, CSIDH	BPR		✓	○	✓
	Dos Santos et al. [DGJ23] (HIC-EKE)	2023	IC, ROM	MLWE, MLWR	UC	✓	✓	○	
	Beguinet et al. [BCP+23] ((O)CAKE)	2023	IC, ROM	MLWE	UC	✓	✓	○	
	Alnahawi [AHHR24] (OCAKE)	2023	IC, ROM	MLWE, LWE	BPR	✓		○	✓
	Pan, Zeng [PZ23] (CAKE)	2023	IC, ROM	LWE	BPR	✓	✓	○	
	Arriaga et al. [ABJS24] (CHIC)	2024	IC, ROM	MLWE	UC	✓		○	✓
	Alnahawi et al. [AASAW24] (NICE-PAKE)	2024	ROM	NLWE / whLWE	BPR	✓ ✓?		○	✓
C2	Zhu, Geng [ZG15]	2015	-	SIDH/CSIDH	CK			○	
	Alsayigh [Als16]	2016	ROM	RLWE	BPR			○	✓
	Ding et al. [DAL+17, Din17] (RLWE-PAK-PPK)	2017	ROM	RLWE	BPR		✓	○	✓
	Gao et al. [GDL+17] (RLWE-PAK-PPK)	2017	ROM	RLWE	BPR			○	✓
	Taraskin et al. [TSJL20] (SIDH-PAK)	2019	ROM	SIDH	BPR			○	✓
	Yang et al. [YGWX19] (RLWE-PAK)	2019	ROM	RLWE	BPR		✓	○	✓
	Jiang et al. [JGH+20] (PAKEs)	2020	ROM	LWE, RLWE	BPR		✓	○	✓
	Ren et al. [RGW23]([RG22]) (MLWE-PAK)	2022	-	MLWE	Hybrid		✓	○	✓
	Seyhan, Akleylek [SA23]	2023	ROM	MLWR	Hybrid			○	✓
	Basu et al. [BSIA23] (MLWR-2PAKA)	2023	ROM	MLWR	DY		✓	○	✓
C3	Katz, Vaikuntanathan [KV09]	2009	CRS	LWE	BPR	✓	✓	○	
	Xu et al. [XHCC17] (RLWE-3PAKE)	2017	ROM	RLWE	BPR			○	✓
	Zhang, Yu [ZY17]	2017	CRS, ROM	LWE	BPR			○	
	Choi et al. [CAK+18] (AtLast)	2018	ROM	RLWE	BPR		✓		○
	Li, Wang [LW18]	2018	CRS	LWE	BPR	✓	✓	○	
	Li, Wang [LW19]	2019	CRS	LWE	BPR	✓	✓	○	
	Karbasi et al. [KAA19] (Ring-PAKE)	2019	CRS	RLWE	BPR	✓	✓	○	
	Yin et al. [YGS+20]	2020	ROM	LWE	BPR		✓	○	○
	Lyu et al. [LLH24]	2024	(Q)ROM	LWE, GA-DDH	UC	✓	✓	✓	○
Augmented									
C2	Gao et al. [GDLL17]	2018	-	RLWE	UC		✓	○	✓
C3	Zhu et al. [ZHS14]	2014	-	SIDH	CK		✓	○	
	Feng et al. [FHZ+18]	2018	ROM	RLWE	BPR		✓	○	✓
	Liu et al. [LZJY19]	2019	ROM	RLWE	Hybrid			○	✓
	Dabra et al. [DBK20] (LBA-PAKE)	2020	ROM	RLWE	FTG		✓	○	✓
	Li et al. [LWM22]	2020	CRS	LWE, LWR	BPR	✓	✓	○	✓
	Tang et al. [TLZ+21]	2021	ROM	RLWE	BPR		✓	○	✓
	Islam, Basu [IB21] (BP-3PAKA)	2021	ROM	RLWE	BPR		✓	○	✓
	Ding et al. [DCQ22]	2022	RoR	RLWE	FTG		✓	○	✓
	Abdalla et al. [AEK+22a] (X-GA-PAKE)	2022	CRS	CSIDH	BPR	✓		○	
	Wang et al. [WCL+23] (LB-ID-2PAKA)	2023	ROM	MLWE	BPR		✓	○	
	Dharminder et al. [DRD+23]	2023	Standard	RLWE	Hybrid		✓	○	✓
	Dadsena et al. [DJRD23]	2023	ROM	RLWE	BPR		✓	○	✓
	Kumar et al. [KGKD23]	2023	ROM	RLWE	BPR		✓	○	✓
	Guo et al. [GSG+23]	2023	ROM	MLWE	BPR		✓	○	✓
	Chaudhary et al. [CKS23]	2023	ROM	RLWE	BPR		✓	○	✓

THANK YOU

Questions?

Nouri Alnahawi - David Haas - Erik Mauß - Alex Wiesmaier
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